

Fuzzy Right (Left) Ideals in Hypergroupoids and Fuzzy Bi-ideals in Hypersemigroups

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Abstract. Most of the results on semigroups or ordered semigroups can be transferred to hypersemigroups or to ordered hypersemigroups, respectively. The same, if we replace the word “semigroup” by “groupoid”, “hypersemigroup” by “hypergroupoid”. We show the way we pass from fuzzy ordered semigroups to fuzzy hypersemigroups.

Key Words: Hypergroupoid, hypersemigroup, fuzzy right (left) ideal, fuzzy bi-ideal, regular, intra-regular

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1 Introduction

In our paper [2] we gave, among others, some equivalent definitions of fuzzy right (left) ideals and fuzzy bi-ideals in ordered semigroups which are very useful for applications. Besides, they show how similar is the theory of ordered semigroups based on fuzzy ideals with the theory of ordered semigroups based on ideals or on ideal elements. Using these definitions many results on fuzzy ordered semigroups or on fuzzy semigroups (without order) can be drastically simplified. The present paper is based on our paper [2], and the aim is to show the way we pass from fuzzy groupoids (semigroups) to fuzzy hypergroupoids (hypersemigroups) or from ordered semigroups to hypersemigroups. We introduce the concepts of fuzzy right and fuzzy left ideals of hypergroupoids and the concept of a fuzzy bi-ideal of an hypersemigroup and we show that a fuzzy subset f of an hypergroupoid H is a fuzzy right (resp. fuzzy left) ideal of H if and only if $f \circ 1 \preceq f$ (resp. $1 \circ f \preceq f$) and for an hypersemigroup H , a fuzzy subset f of H is a bi-ideal of H if and only if $f \circ 1 \circ f \preceq f$. In addition to its aim we could mention that the present paper together with the paper in [3] show that these characterizations are very useful for the investigation exactly as in the case of ordered semigroups.

2 Main results

An *hypergroupoid* is a nonempty set H with an hyperoperation

$$\circ : H \times H \rightarrow \mathcal{P}^*(H) \mid (a, b) \rightarrow a \circ b \text{ on } H \text{ and an operation}$$

$*$: $\mathcal{P}^*(H) \times \mathcal{P}^*(H) \rightarrow \mathcal{P}^*(H) \mid (A, B) \rightarrow A * B$ on $\mathcal{P}^*(H)$ (induced by the operation of H) such that $A * B = \bigcup_{(a,b) \in A \times B} (a \circ b)$ for every $A, B \in \mathcal{P}^*(H)$

($\mathcal{P}^*(H)$ being the set of nonempty subsets of H).

The operation “ $*$ ” is well defined. Indeed: If $(A, B) \in \mathcal{P}^*(H) \times \mathcal{P}^*(H)$, then $A * B = \bigcup_{(a,b) \in A \times B} (a \circ b)$. For every $(a, b) \in A \times B$, we have $(a, b) \in$

$H \times H$, then $(a \circ b) \in \mathcal{P}^*(H)$, thus we get $A * B \in \mathcal{P}^*(H)$. If $(A, B), (C, D) \in \mathcal{P}^*(H) \times \mathcal{P}^*(H)$ such that $(A, B) = (C, D)$, then $A * B = \bigcup_{(a,b) \in A \times B} (a \circ b) =$

$$\bigcup_{(a,b) \in C \times D} (a \circ b) = C * D.$$

As the operation “ $*$ ” depends on the hyperoperation “ \circ ”, an hypergroupoid can be also denoted by (H, \circ) instead of $(H, \circ, *)$.

If H is an hypergroupoid then, for any $x, y \in H$, we have $x \circ y = \{x\} * \{y\}$. Indeed,

$$\{x\} * \{y\} = \bigcup_{u \in \{x\}, v \in \{y\}} u \circ v = x \circ y.$$

An hypergroupoid H is called *hypersemigroup* if

$$(x \circ y) * \{z\} = \{x\} * (y \circ z) \text{ for every } x, y, z \in H.$$

Since $x \circ y = \{x\} * \{y\}$ for any $x, y \in H$, an hypergroupoid H is an hypersemigroup if and only if, for any $x, y, z \in H$, we have $(\{x\} * \{y\}) * \{z\} = \{x\} * (\{y\} * \{z\})$.

Following Zadeh, if (H, \circ) is an hypergroupoid, we say that f is a fuzzy subset of H (or a fuzzy set in H) if f is a mapping of H into the real closed interval $[0, 1]$ of real numbers, that is $f : H \rightarrow [0, 1]$. For an element a of H , we denote by A_a the subset of $H \times H$ defined as follows:

$$A_a := \{(y, z) \in H \times H \mid a \in y \circ z\}.$$

For two fuzzy subsets f and g of H , we denote by $f \circ g$ the fuzzy subset of H defined by:

$$f \circ g : H \rightarrow [0, 1] \quad a \rightarrow \begin{cases} \bigvee_{(y,z) \in A_a} \min\{f(y), g(z)\} & \text{if } A_a \neq \emptyset \\ 0 & \text{if } A_a = \emptyset. \end{cases}$$

Denote by $F(H)$ the set of all fuzzy subsets of H and by “ \preceq ” the order relation on $F(H)$ defined by:

$$f \preceq g \iff f(x) \leq g(x) \text{ for every } x \in H.$$

We finally show by 1 the fuzzy subset of H defined by: $1 : H \rightarrow [0, 1] \mid x \rightarrow 1(x) := 1$. Clearly, the fuzzy subset 1 is the greatest element of the ordered set $(F(H), \preceq)$ (that is, $1 \succeq f \forall f \in F(H)$). For two fuzzy subsets f and g of an hypergroupoid H we denote by $f \wedge g$ the fuzzy subset of H defined as follows:

$$f \wedge g : H \rightarrow [0, 1] \mid x \rightarrow (f \wedge g)(x) := \min\{f(x), g(x)\}.$$

One can easily prove that the fuzzy subset $f \wedge g$ is the infimum of the fuzzy subsets f and g , so we write $f \wedge g = \inf\{f, g\}$.

We denote the hyperoperation on H and the multiplication between the two fuzzy subsets of H by the same symbol (no confusion is possible).

The Proposition 1 below, though clear, plays an essential role in the theory of hypergroupoids.

Proposition 1. *If (H, \circ) is an hypergroupoid, $x \in H$ and $A, B \in \mathcal{P}^*(H)$, then*

$$x \in A * B \iff x \in a \circ b \text{ for some } a \in A, b \in B.$$

Lemma 2. *Let H be an hypergroupoid. Then we have the following:*

- (1) *If $A, B, C \in \mathcal{P}^*(H)$ such that $A \subseteq B$, then $A * C \subseteq B * C$ and $C * A \subseteq C * B$.*
- (2) *$H * H \subseteq H$.*

Definition 3. Let H be an hypergroupoid. A fuzzy subset f of H is called a *fuzzy right ideal* of H if

$$f(x \circ y) \geq f(x) \text{ for every } x, y \in H,$$

in the sense that if $x, y \in H$ and $u \in x \circ y$, then $f(u) \geq f(x)$.

Theorem 4. *Let H be an hypergroupoid and f a fuzzy subset of H . Then f is a fuzzy right ideal of H if and only if*

$$f \circ 1 \preceq f.$$

Proof. \implies . Let $a \in H$. Then $(f \circ 1)(a) \leq f(a)$. In fact:

- (i) If $A_a = \emptyset$, then $(f \circ 1)(a) := 0$. Since f is a fuzzy subset of H , we have $f(a) \geq 0$. Thus we have $(f \circ 1)(a) \leq f(a)$.
- (ii) Let $A_a \neq \emptyset$. Then

$$(f \circ 1)(a) := \bigvee_{(y,z) \in A_a} \min\{f(y), 1(z)\} = \bigvee_{(y,z) \in A_a} f(y) \quad (*)$$

On the other hand, $f(y) \leq f(a)$ for every $(y, z) \in A_a$ (**)
Indeed, if $(y, z) \in A_a$, then $a \in y \circ z$ and, since f is a fuzzy right ideal of H , we have $f(a) \geq f(y)$.

By (**), we have $\bigvee_{(y,z) \in A_a} f(y) \leq f(a)$. Then, by (*), $(f \circ 1)(a) \leq f(a)$.

\Leftarrow . Let $x, y \in H$ and $u \in x \circ y$. Then $f(u) \geq f(x)$. Indeed:
Since $x, y \in H$ and $u \in x \circ y$, we have $(x, y) \in A_u$. Since $A_u \neq \emptyset$, we have

$$(f \circ 1)(u) := \bigvee_{(t,s) \in A_u} \min\{f(t), 1(s)\} = \bigvee_{(t,s) \in A_u} f(t) \geq f(x) \quad \forall (t, s) \in A_u.$$

Since $(x, y) \in A_u$, we have $(f \circ 1)(u) \geq f(x)$. Since $u \in x \circ y \subseteq H$, we have $u \in H$. Since $u \in H$, by hypothesis, we have $(f \circ 1)(u) \leq f(u)$. Then we have $f(u) \geq f(x)$. \square

Definition 5. Let H be an hypergroupoid. A fuzzy subset f of H is called a *fuzzy left ideal* of H if

$$f(x \circ y) \geq f(y) \text{ for every } x, y \in H,$$

in the sense that if $x, y \in H$ and $u \in x \circ y$, then $f(u) \geq f(y)$.

In a similar way as in Theorem 4, we can prove the following theorem.

Theorem 6. A fuzzy subset f of an hypergroupoid H is a fuzzy left ideal of H if and only if, for any fuzzy subset f of H , we have

$$1 \circ f \preceq f.$$

Definition 7. Let H be an hypergroupoid. A fuzzy subset f of H is called a *fuzzy quasi-ideal* of H if

$$x \in b \circ s \text{ and } x \in t \circ c \implies f(x) \geq \min\{f(b), f(c)\} \quad \forall x, b, s, t, c \in H.$$

Theorem 8. Let H be an hypergroupoid. A fuzzy subset f of H is a fuzzy quasi-ideal of H if and only if

$$(f \circ 1) \wedge (1 \circ f) \preceq f.$$

Proof. \implies . Let $x \in H$. Then $\left((f \circ 1) \wedge (1 \circ f)\right)(x) \leq f(x)$, that is $\min\{(f \circ 1)(x), (1 \circ f)(x)\} \leq f(x)$. Indeed: For $A_x = \emptyset$ this is clear. Let $A_x \neq \emptyset$. Then

$$(f \circ 1)(x) := \bigvee_{(y,s) \in A_x} \min\{f(y), 1(s)\} = \bigvee_{(y,s) \in A_x} f(y)$$

and

$$(1 \circ f)(x) := \bigvee_{(t,z) \in A_x} \min\{1(t), f(z)\} = \bigvee_{(t,z) \in A_x} f(z).$$

If $f(x) \geq (f \circ 1)(x)$ then, since $(f \circ 1)(x) \geq \min\{(f \circ 1)(x), (1 \circ f)(x)\}$, we have

$$f(x) \geq \min\{(f \circ 1)(x), (1 \circ f)(x)\}.$$

Let $f(x) < (f \circ 1)(x)$. Then there exists $(y, s) \in A_x$ such that $f(y) > f(x)$ (*) (otherwise $(f \circ 1)(x) \leq f(x)$ which is impossible).

We prove that $f(x) \geq f(z)$ for every $(t, z) \in A_x$. Then we have

$$f(x) \geq \bigvee_{(t,z) \in A_x} f(z) = (1 \circ f)(x) \geq \min\{(f \circ 1)(x), (1 \circ f)(x)\}$$

and the proof is complete.

Let now $(t, z) \in A_x$. Then $f(x) \geq f(z)$. Indeed: Since $(y, s) \in A_x$, we have $y, s \in H$ and $x \in y \circ s$. Since $(t, z) \in A_x$, we have $t, z \in H$ and $x \in t \circ z$. Since $x, y, s, t, z \in H$ such that $x \in y \circ s$ and $x \in t \circ z$, by hypothesis, we have $f(x) \geq \min\{f(y), f(z)\}$. If $\min\{f(y), f(z)\} = f(y)$, then $f(x) \geq f(y)$ which is impossible by (*). Thus we have $\min\{f(y), f(z)\} = f(z)$, and $f(x) \geq f(z)$.

\Leftarrow . Let $x, b, s, t, c \in H$ such that $x \in b \circ s$ and $x \in t \circ c$. Then $f(x) \geq \min\{f(b), f(c)\}$. Indeed: By hypothesis, we have

$$f(x) \geq \left((f \circ 1) \wedge (1 \circ f) \right)(x) := \min\{(f \circ 1)(x), (1 \circ f)(x)\}.$$

Since $x \in b \circ s$, we have $(b, s) \in A_x$, then

$$(f \circ 1)(x) := \bigvee_{(u,v) \in A_x} \min\{f(u), 1(v)\} = \bigvee_{(u,v) \in A_x} f(u) \geq f(b).$$

Since $x \in t \circ c$, we have $(t, c) \in A_x$, then

$$(1 \circ f)(x) := \bigvee_{(w,k) \in A_x} \min\{1(w), f(k)\} = \bigvee_{(w,k) \in A_x} f(k) \geq f(c).$$

Thus we have

$$f(x) \geq \min\{(f \circ 1)(x), (1 \circ f)(x)\} \geq \min\{f(b), f(c)\}.$$

□

The proof of the associativity of fuzzy sets on semigroups given in [1] can be naturally transferred to hypersemigroups in the following proposition.

Proposition 9. *If H is an hypersemigroup, then the set of all fuzzy subsets of H is a semigroup.*

Proof. Let f, g, h be fuzzy subsets of H and $a \in H$. Then

$$\left((f \circ g) \circ h \right)(a) = \left(f \circ (g \circ h) \right)(a).$$

Indeed: If $A_a = \emptyset$, then $\left((f \circ g) \circ h \right)(a) = 0 = \left(f \circ (g \circ h) \right)(a)$.

Let $A_a \neq \emptyset$. Then

$$\left((f \circ g) \circ h\right)(a) = \bigvee_{(y,z) \in A_a} \min\{(f \circ g)(y), h(z)\}$$

and

$$\left(f \circ (g \circ h)\right)(a) = \bigvee_{(u,v) \in A_a} \min\{f(u), (g \circ h)(v)\}.$$

We put

$$t := \bigvee_{(y,z) \in A_a} \min\{(f \circ g)(y), h(z)\} \quad (*)$$

$$s := \bigvee_{(u,v) \in A_a} \min\{f(u), (g \circ h)(v)\}.$$

We prove that $t \geq \min\{f(u), (g \circ h)(v)\}$ for every $(u, v) \in A_a$. Then we have $t \geq s$. In a similar way we prove that $s \geq t$, and so $s = t$.

Let $(u, v) \in A_a \Rightarrow t \geq \min\{f(u), (g \circ h)(v)\}$?)

(A) Let $t \geq f(u)$. Since $f(u) \geq \min\{f(u), (g \circ h)(v)\}$, we have

$$t \geq \min\{f(u), (g \circ h)(v)\}.$$

(B) Let $t < f(u)$. We consider the cases:

(a) Let $A_v = \emptyset$. Then $(g \circ h)(v) := 0$. Since f is a fuzzy set in H , we have $f(u) \geq 0$, then $\min\{f(u), (g \circ h)(v)\} = 0$. Since $t \in [0, 1]$, we have $t \geq 0$, so $t \geq \min\{f(u), (g \circ h)(v)\}$.

(b) Let $A_v \neq \emptyset$. Then $(g \circ h)(v) = \bigvee_{(c,d) \in A_v} \min\{g(c), h(d)\}$. We prove that

$$t \geq \min\{g(c), h(d)\} \text{ for every } (c, d) \in A_v.$$

Then we have $t \geq (g \circ h)(v) \geq \min\{f(u), (g \circ h)(v)\}$.

Let now $(c, d) \in A_v \Rightarrow t \geq \min\{g(c), h(d)\}$?)

(i) Let $t \geq g(c)$. Since $g(c) \geq \min\{g(c), h(d)\}$, we have $t \geq \min\{g(c), h(d)\}$.

(ii) Let $t < g(c)$. Since $(u, v) \in A_a$, we have $a \in u \circ v$. Since $(c, d) \in A_v$, we have $v \in c \circ d$. Then we have

$$\begin{aligned} a \in u \circ v &= \{u\} * \{v\} \subseteq \{u\} * (c \circ d) \text{ (by Lemma 2)} \\ &= (u \circ c) * \{d\} \text{ (the operation “*” is associative).} \end{aligned}$$

By Proposition 1, $a \in w \circ d$ for some $w \in u \circ c$. Since $a \in w \circ d$, we have $(w, d) \in A_a$ then, by (*), we have

$$t \geq \min\{(f \circ g)(w), h(d)\} \quad (**)$$

Since $w \in u \circ c$, we have $(u, c) \in A_w$, then A_w is a nonempty set and we have

$$(f \circ g)(w) = \bigvee_{(l,k) \in A_w} \min\{f(l), g(k)\} \geq \min\{f(u), g(c)\} \quad (***)$$

Since $t < f(u)$ and $t < g(c)$, we have $t < \min\{f(u), g(c)\}$. Then, by $(***)$, $t < (f \circ g)(w)$. Then, by $(**)$, $t \geq h(d)$. Since $h(d) \geq \min\{g(c), h(d)\}$, we have $t \geq \min\{g(c), h(d)\}$ and the proof of the proposition is complete. \square

According to Proposition 9, for any fuzzy subsets f, g, h of H , we write

$$(f \circ g) \circ h = f \circ (g \circ h) := f \circ g \circ h.$$

Definition 10. Let H be an hypersemigroup. A fuzzy subset f of H is called a *fuzzy bi-ideal* of H if

$$f\left((x \circ y) * \{z\}\right) \geq \min\{f(x), f(z)\} \quad \forall x, y, z \in H,$$

in the sense that if $u \in (x \circ y) * \{z\}$, then $f(u) \geq \min\{f(x), f(z)\}$.

Theorem 11. Let H be an hypersemigroup and f be a fuzzy subset of S . Then f is a fuzzy bi-ideal of H if and only if

$$f \circ 1 \circ f \preceq f.$$

Proof. \implies . Let $a \in H$. Then $(f \circ 1 \circ f)(a) \leq f(a)$. In fact: If $A_a = \emptyset$, then

$$(f \circ 1 \circ f)(a) = \left((f \circ 1) \circ f\right)(a) := 0 \leq f(a).$$

Let $A_a \neq \emptyset$. Then

$$(f \circ 1 \circ f)(a) := \bigvee_{(y,z) \in A_a} \min\{(f \circ 1)(y), f(z)\}.$$

It is enough to prove that

$$\min\{(f \circ 1)(y), f(z)\} \leq f(a) \quad \text{for every } (y, z) \in A_a \quad (*)$$

For this purpose, let $(y, z) \in A_a$. If $A_y = \emptyset$, then $(f \circ 1)(y) := 0 \leq f(z)$, and $\min\{(f \circ 1)(y), f(z)\} = 0 \leq f(a)$. Let now $A_y \neq \emptyset$. Then

$$(f \circ 1)(y) := \bigvee_{(x,w) \in A_y} \min\{f(x), 1(w)\} = \bigvee_{(x,w) \in A_y} f(x) \quad (1)$$

We consider the following cases:

(i) Let $f(a) \geq (f \circ 1)(y)$. Then

$$f(a) \geq (f \circ 1)(y) \geq \min\{(f \circ 1)(y), f(z)\},$$

and condition (*) is satisfied.

(ii) Let $f(a) < (f \circ 1)(y)$. Then there exists $(x, w) \in A_y$ such that $f(a) < f(x)$ (2) Indeed: If

$f(a) \geq f(x)$ for every $(x, w) \in A_y$, then $f(a) \geq \bigvee_{(x,w) \in A_y} f(x)$. Then, by (1),

we get $f(a) \geq (f \circ 1)(y)$ which is impossible.

Since $(y, z) \in A_a$, we have $y, z \in H$ and $a \in y \circ z$. Since $(x, w) \in A_y$, we have $x, w \in H$ and $y \in x \circ w$. Then we have

$$a \in y \circ z = \{y\} * \{z\} \subseteq (x \circ w) * \{z\}$$

and, since f is a fuzzy bi-ideal of H , we have

$$f(a) \geq \min\{f(x), f(z)\} \quad (3)$$

If $f(x) \leq f(z)$ then $f(a) \geq f(x)$, which is impossible by (2). Thus we have

$$f(x) > f(z) \quad (4)$$

Then $\min\{f(x), f(z)\} = f(z)$ and, by (3), $f(a) \geq f(z)$ (5)

Since $(x, w) \in A_y$, by (1), we have $f(x) \leq (f \circ 1)(y)$ (6)

By (4) and (6), we have $f(z) < f(x) \leq (f \circ 1)(y)$, then

$$\min\{f \circ 1)(y), f(z)\} = f(z) \leq f(a) \text{ (by (5))},$$

and condition (*) is satisfied.

\Leftarrow . Let $x, y, z \in H$ and $u \in (x \circ y) * \{z\}$. Then $f(u) \geq \min\{f(x), f(z)\}$.

Indeed:

Since $u \in (x \circ y) * \{z\}$, by Proposition 1, there exists $a \in x \circ y$ such that $u \in a \circ z$. Since $a \in x \circ y$, we have $(x, y) \in A_a$. Since $u \in a \circ z$, we have $(a, z) \in A_u$. Since $(a, z) \in A_u$, A_u is a nonempty set and we also have

$$(f \circ 1 \circ f)(u) = \bigvee_{(t,s) \in A_u} \min\{(f \circ 1)(t), f(s)\} \geq \min\{(f \circ 1)(a), f(z)\}.$$

Since $(x, y) \in A_a$, we have $A_a \neq \emptyset$ and we also have

$$(f \circ 1)(a) = \bigvee_{(z,w) \in A_a} \min\{f(z), 1(w)\} \geq \min\{f(x), 1(y)\} = f(x).$$

Then we have

$$(f \circ 1 \circ f)(u) \geq \min\{f(x), f(z)\}.$$

On the other hand, since $u \in (x \circ y) * \{z\} \subseteq H * H \subseteq H$ and $f \circ 1 \circ f \preceq f$, we have $(f \circ 1 \circ f)(u) \leq f(u)$. Thus we have

$$f(u) \geq (f \circ 1 \circ f)(u) \geq \min\{f(x), f(z)\},$$

and the proof of the theorem is complete. \square

Note. The characterization of fuzzy right (left) and fuzzy bi-ideals of an hypersemigroup H using the greatest element 1 of the ordered set of fuzzy subsets of H is very useful for further investigation. Using these definitions many proofs on hypersemigroups can be drastically simplified. As an example, one can immediately see that every fuzzy right (or fuzzy left) ideal of H is a fuzzy bi-ideal of H . Indeed, if f is a fuzzy right ideal of H , then $f \circ 1 \circ f = f \circ (1 \circ f) \preceq f \circ 1 \preceq f$. Let us give two more examples to clarify what we say. Further interesting information concerning this structure will be given in a forthcoming paper. We begin with the definition of regular hypersemigroup. An hypersemigroup H is called *regular* if for every $a \in H$ there exists $x \in H$ such that $a \in (a \circ x) * \{a\}$. An hypersemigroup H is called *intra-regular* if for every $a \in H$ there exist $x, y \in H$ such that $a \in (x \circ a) * (a \circ y)$. An hypersemigroup H is regular if and only if, for every fuzzy subset f of H , we have $f \preceq f \circ 1 \circ f$. It is intra-regular if and only if, for every fuzzy subset f of H , we have $f \preceq 1 \circ f \circ f \circ 1$ [3]. If H is a regular hypersemigroup and f is a fuzzy bi-ideal of H , then there exist a fuzzy right ideal h and a fuzzy left ideal g of H such that $f = h \circ g$. In fact, we have

$$f = f \circ 1 \circ f = f \circ 1 \circ (f \circ 1 \circ f) \preceq (f \circ 1) \circ (1 \circ f) \preceq f \circ 1 \circ f = f.$$

So $f = (f \circ 1) \circ (1 \circ f)$, where $f \circ 1$ is a fuzzy right and $1 \circ f$ is a fuzzy left ideal of H . If H is an intra-regular hypersemigroup then, for every fuzzy right ideal f and every fuzzy left ideal g of H , we have $f \wedge g \preceq g \circ f$. Indeed, we have

$$f \wedge g \preceq 1 \circ (f \wedge g) \circ (f \wedge g) \circ 1 \preceq (1 \circ g) \circ (f \circ 1) \preceq g \circ f.$$

In a similar way, using the results of the present paper one can immediately prove that if H is a regular hypersemigroup then, for every fuzzy right ideal f and every fuzzy left ideal g of H , we have $f \wedge g = f \circ g$. \square

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