Armenian Journal of Mathematics
Volume 16, Number 12, 2024, 1–10
https://doi.org/10.52737/18291163-2024.16.12-1-10

Implicit Elliptic Problems with p-Laplacian

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Abstract. In this research, we will study the existence of weak solutions for a class of implicit elliptic equations involving the p-Laplace operator. Using a Krasnoselskii–Schaefer type theorem we establish our result, extending and complementing those obtained by R. Precup, 2020, and Marino and Paratore, 2021.

Key Words: Implicit Elliptic Problems, Krasnoselskii Theorem, p-Laplacian
Mathematics Subject Classification 2020: 35J92, 47H10, 47J05

Introduction

In this article we focus on the following boundary elliptic problem:

$$\begin{gathered} -\triangle_p u = f(x, u, \nabla u, \triangle_p u) + g(x, u, \nabla u) \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \Gamma, \end{gathered} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary Γ, $(n \geq 3)$, $\triangle_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian, $2 < p < +\infty$, $f : \Omega \times \mathbb{R} \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ and $g : \Omega \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ are Carathéodory functions.

In 1958, Krasnoselskii proved his famous result on the existence of fixed point for a sum of two operators, one of which is is a contraction and the second one is compact, defined in a convex and closed set, and concluding that its sum has a fixed point. Since then, many extensions have emerged with various types of generalized contractions and generalized compact operators, which are generally applied to the resolution of specific problems posed in natural sciences and physics. In particular, his result gives a method for solving Dirichlet problems in which nonlinear sources can be expressed by the sum of two terms to which appropriate restrictions are imposed to fulfill the hypotheses in Krasnoselskii's theorem. Precup [10] studied the Dirichlet problem with the Laplacian operator ($p = 2$) for implicit equations involving two sources, one source containing the Laplacian and another containing the gradient, via a Krasnoselskii-type fixed point theorem and suggested the application of his technique to general elliptic operators that replace the Laplacian and to other classes of implicit differential equations. Thus, inspired by the ideas introduced by Precup, this paper aims to study the existence of solutions for the implicit equation (1) involving the p-Laplacian with $p > 2$. This extension is not trivial due to the mathematical difficulties posed by the degenerate quasilinear elliptic operator, compared to the Laplacian operator: the lack of Hilbert structure of the domain of the operator, the absence of linearity and the complicated spectral properties. We point out that implicit elliptic equations have been intensively studied in the literature (see [1, 2, 3, 5, 8, 7] and references therein), and have multiple applications to the calculus of variations, nonlinear elasticity, problems of phase transitions and optimal design (see, e.g., [4]).

The paper is organized as follows. In Section 1, as preliminaries, we recall some properties of the inverse operator of p-Laplacian and the main tool, a hybrid theorem of Krasnoselskii type due to Gao et al.[6]. Section 2 is devoted to state and prove our main result about existence of weak solutions for problem (1).

1 Preliminaries

Let $W_0^{1,p}(\Omega)$, $(1 < p)$, be the usual Sobolev space equipped with the norm

$$\|u\| = \left(\int_\Omega |u|^p\right)^{1/p}, \quad u \in W_0^{1,p}(\Omega),$$

and $\|u\|_p = \left(\int_\Omega |u|^p\right)^{1/p}$ denotes the norm in $L^p(\Omega)$.

By the Sobolev embedding theorem, for any $1 \leq \theta \leq p^*$ $(1 \leq \theta < p^*)$, $p < N$, the embedding $W_0^{1,p}(\Omega) \hookrightarrow L^\theta(\Omega)$ is continuous (compact) and there exists a positive constant C_θ such that $\|u\|_\theta \leq C_\theta \|u\|$ for all $u \in W_0^{1,p}(\Omega)$.

Consider the first eigenvalue λ_1 of the problem

$$\begin{cases} -\triangle_p u = \lambda_1 |u|^{p-2} u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Thanks to the work of Peral [9], one has that

$$\lambda_1 := \inf_{u \in W_0^{1,p}(\Omega)\setminus\{0\}} \frac{\int_\Omega |\nabla u|^p \, dx}{\int_\Omega |u|^p \, dx}$$

is isolated and simple, also its corresponding first eigenfunction is positive. Thus, the best(smallest) embedding constant for the inclusion $W_0^{1,p}(\Omega) \hookrightarrow L^p(\Omega)$ is $1/\sqrt[p]{\lambda_1}$.

Let $W^{-1,p'}(\Omega)$ be the dual space of $W_0^{1,p}(\Omega)$. Also, an embedding constant for the inclusion $L^{p'}(\Omega) \hookrightarrow W^{-1,p'}(\Omega)$ is $1/\sqrt[p]{\lambda_1}$.

It is well known, that the problem

$$\begin{cases} -\triangle_p u = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

has a unique weak solution $u \in W_0^{1,p}(\Omega)$ for $f \in W^{-1,p'}(\Omega)$. Thus, $S = -\triangle_p : W_0^{1,p}(\Omega) \to W^{-1,p'}(\Omega)$ has the following properties:

(i) S is bijective and uniformly continuous on bounded sets.

(ii) The operator $S^{-1} : W^{-1,p'}(\Omega) \to W_0^{1,p}(\Omega)$ is continuous and for any $v_1, v_2 \in W^{-1,p'}(\Omega)$, the following estimate holds

$$\|S^{-1}v_1 - S^{-1}v_2\| \leq M_1^{1/(p-1)} \|v_1 - v_2\|_{-1}^{1/(p-1)} \tag{2}$$

for some constant $M_1 > 0$ independent of v_1 and v_2.

(iii)

$$\|Su\|_{-1} = \|u\|^{p-1}, \quad u \in W_0^{1,p}(\Omega), \tag{3}$$

where $\|.\|_{-1}$ denotes the norm in $W^{-1,p'}(\Omega)$, $1/p + 1/p' = 1$.

We recall that our approach is based on a extension of Krasnoselskii's theorem, which combine Banach's contraction principle with Schaefer's fixed point theorem due to Gao, Li and Zhang [6], and on the previous mentioned work [10] by Precup.

Theorem 1 (Gao-Li-Zhang) *Let D_R be a closed ball centered at the origin and of radius R of a Banach space X, and let A, B be operators such that*

(i) $A : X \to X$ *is a contraction;*

(ii) $B : D_R \to X$ *is continuous with* $B(D_R)$ *relatively compact;*

(iii) $x \neq A(x) + \lambda B(x)$ *for all* $x \in \partial D_R$ *and* $\lambda \in]0, 1[$.

Then the operator $A+B$ has at least one fixed point, i.e., there exits $x \in D_R$ such that

$$x = A(x) + B(x).$$

Remark 1 In practice, we use the method of a priori estimates, so both operators A, B are defined on the whole space X, and a ball D_R as required by condition (iii) of Theorem 1 exists if the set

$$Y = \{x \in X | x = A(x) + \lambda B(x), \quad \text{for some } \lambda \in [0, 1]\}$$

is bounded in X.

Lemma 1 *For any* $(p^*)' \le \tau \le p$, *the embeddings*

$$W_0^{1,p}(\Omega) \hookrightarrow L^\tau(\Omega), \quad L^p(\Omega) \hookrightarrow L^\tau(\Omega), \quad L^\tau(\Omega) \hookrightarrow W^{-1,p'}(\Omega)$$

are continuous, and we may consider positive constants c_1, c_2, c_3 *such that*

$$\|u\|_\tau \le c_1\|u\|, \quad \|u\|_\tau \le c_2\|u\|_p, \quad \|u\|_{-1} \le c_3\|u\|_\tau, \tag{4}$$

with

$$c_2 = c_1\sqrt[p]{\lambda_1}, \quad c_3 = \frac{c_\Omega}{c_1\lambda_1^{2/p}}, \quad \textit{and} \quad c_\Omega = |\Omega|^{(p-2)/p}.$$

Proof. From (4), we get

$$\|u\|_\tau \le c_2\|u\|_p \le \frac{c_2}{\sqrt[p]{\lambda_1}}\|u\|, \quad \text{for } u \in W_0^{1,p}(\Omega),$$

which give us $c_2 = c_1\sqrt[p]{\lambda_1}$.

On the other hand, in view of the inclusions $L^{p'}(\Omega) \hookrightarrow W^{-1,p'}(\Omega)$, $L^p(\Omega) \hookrightarrow L^{p'}(\Omega)$, we have for $u \in W_0^{1,p}(\Omega)$,

$$\|u\|_{-1} \le \frac{1}{\sqrt[p]{\lambda_1}}\|u\|_{p'} \le \frac{c_\Omega}{\sqrt[p]{\lambda_1}}\|u\|_p \le \frac{c_\Omega}{\lambda_1^{2/p}}\|u\|.$$

Now, since

$$\|u\|_{-1} \le c_3\|u\|_\tau \le c_3c_1\|u\|,$$

it follows that $c_1c_3 = c_\Omega/\lambda_1^{2/p}$. $\square$

Setting $v = Su$, equation (1) is equivalent to the fixed point equation

$$v = f\left(x, S^{-1}v, \nabla S^{-1}v, -v\right) + g\left(x, S^{-1}v, \nabla S^{-1}v\right), \tag{5}$$

which will be solved in the Lebesgue space $L^\tau(\Omega)$ with $\tau \ge (p^*)'$.

Define operators $A, B : L^\tau(\Omega) \to L^\tau(\Omega)$ by

$$Av = f\left(\cdot, S^{-1}v, \nabla S^{-1}v, -v\right), \qquad Bv = g\left(\cdot, S^{-1}v, \nabla S^{-1}v\right).$$

Then equation (5) becomes the operator equation

$$v = A(v) + B(v).$$

Our idea is to use Theorem 1 to find the fixed point for the sum $A + B$ in $W_0^{1,p}(\Omega)$. For this goal, we need to impose additional conditions on f and g to guarantee that the two operators are well defined from $L^\tau(\Omega)$ to itself, and then, we will show that A is a contraction, and B is completely continuous. We conclude, by establishing a priori bounds for the solutions to the problem as required by Remark 1.

2 Existence of Solutions

In this section, we present our main result. More precisely, under suitable conditions, we prove the existence of a solution to problem (1) by applying Theorem 1.

First, we give the following hypotheses on f and g.

(A_1) There exist $a, b, c \geq 0$ such that

$$|f(x,y,z,w) - f(x,\overline{y},\overline{z},\overline{w})| \leq a|y-\overline{y}|^{p-1} + b|z-\overline{z}|^{p-1} + c|w-\overline{w}|,$$
$$f(\cdot\,,0,0,0) \in L^p(\Omega).$$

(A_2) There exist constants $a_0, b_0 \geq 0$, $\alpha \in [1, p^*/(p^*)']$, $\beta \in [1, p/(p^*)']$, and $h \in L^p(\Omega)$ such that

$$|g(x,y,z)| \leq a_0|y|^\alpha + b_0|z|^\beta + h(x) \quad \text{for any } y \in \mathbb{R}, z \in \mathbb{R}^n \text{ and } a.e.\ x \in \Omega.$$

(A_3) $yg(x,y,z) \leq \sigma|y|^p$ for all $y \in \mathbb{R}$, $z \in \mathbb{R}^n$, $a.e.$ $x \in \Omega$, and some $\sigma < \sigma_0\lambda_1$, $0 < \sigma_0 < 1$, where λ_1 is the first eigenvalue of $(-\Delta_p, W_0^{1,p}(\Omega))$.

(A_4) $\ell_0 := \left(\dfrac{a}{\lambda_1^{2/p}} + \dfrac{b}{\lambda_1^{(3-p)/p}}\right)\left(c_1|\Omega|^{\frac{1}{p}-\frac{1}{\tau}}\right)^{p-2} M_1 + c$, $\ell_1 := \dfrac{a}{\lambda_1} + \dfrac{bc_\Omega}{\lambda_1^{(1)/p}} + c$, $\ell = \max\{\ell_0, \ell_1\} < 1$, $\sigma_0 = 1 - \ell$.

We are now ready to state our main result.

Theorem 2 *Let $(p^*)' \leq \tau \leq p$. Assume that assumptions (A_1)–(A_4) hold true. Then* (1) *has at least one solution $u \in W_0^{1,p}(\Omega)$ with $\Delta_p u \in L^\tau(\Omega)$.*

For the proof of this theorem, we need to establish the following three lemmas.

Lemma 2 *Suppose that (A_1) and (A_4) hold. Then A is a contraction on $L^\tau(\Omega)$, $\tau \in [1, p/(p-1)]$, provided a and b are sufficiently small.*

Proof. The Carathéodory conditions ensure that for every measurable function $v \in L^\tau(\Omega)$, the function $f\left(\cdot, S^{-1}v, \nabla S^{-1}v, -v\right)$ is also measurable. Furthermore,

$$\begin{aligned}&\left\|f\left(\cdot, S^{-1}v, \nabla S^{-1}v, -v\right)\right\|_\tau = \left\|f\left(\cdot, S^{-1}v, \nabla S^{-1}v, -v\right) - f(\cdot,0,0,0)\right\|_\tau \\ &\leq a\left\||S^{-1}v|^{p-1}\right\|_\tau + b\left\||\nabla S^{-1}v|^{p-1}\right\|_\tau + c\|v\|_\tau \end{aligned} \tag{6}$$

But, using the inequalities

$$\|z\|_{\tau(p-1)} \leq c_\tau\|z\| \quad \text{for all } z \in W_0^{1,p}(\Omega), \tag{7}$$

and

$$\|z\|_{\tau(p-1)} \leq c_p \|z\|_p \quad \text{for all } z \in L^p(\Omega), \tag{8}$$

where c_τ and c_p are the best constants for the embeddings $W_0^{1,p}(\Omega) \hookrightarrow L^{\tau(p-1)}(\Omega)$ and $L^p(\Omega) \hookrightarrow L^{\tau(p-1)}(\Omega)$, respectively, we have for $v \in L^\tau(\Omega)$,

$$\begin{aligned}\left\||S^{-1}v|^{p-1}\right\|_\tau \leq \left\||S^{-1}v|\right\|_{\tau(p-1)}^{p-1} \leq c_\tau^{p-1}\|S^{-1}v\|^{p-1} &= c_\tau^{p-1}\|v\|_{-1} \\ &\leq c_\tau^{p-1} c_3 \|v\|_\tau < \infty,\end{aligned}$$

and, similarly,

$$\left\||\nabla S^{-1}v|^{p-1}\right\|_\tau \leq c_p^{p-1} c_3 \|v\|_\tau < \infty.$$

So, from (6) and the above inequalities, A is well defined from $L^\tau(\Omega)$ to itself.

Furthermore, we can use this last process to obtain

$$\begin{aligned}\|Av - Aw\|_\tau &\leq a\left\|S^{-1}v - S^{-1}w\right\|_{\tau(p-1)}^{p-1} + b\left\|\nabla S^{-1}v - \nabla S^{-1}w\right\|_{\tau(p-1)}^{p-1} \\ &\quad + c\|v-w\|_\tau \\ &\leq ac_\tau^{p-1}M_1c_3\|v-w\|_\tau + bc_p^{p-1}M_1c_3\|v-w\|_\tau + c\|v-w\|_\tau \\ &\leq \left[(ac_\tau^{p-1} + bc_p^{p-1})M_1c_3 + c\right]\|v-w\|_\tau \\ &= \left[\left(\frac{a}{\lambda_1^{2/p}} + \frac{b}{\lambda_1^{(3-p)/p}}\right)\left(c_1|\Omega|^{1/p-1/\tau}\right)^{p-2} M_1 + c\right]\|v-w\|_\tau.\end{aligned}$$

It follows from hypothesis (A_4), that A is a contraction. $\square$

Lemma 3 *Suppose that (A_2) is satisfied. Then the operator $B : L^\tau(\Omega) \longrightarrow L^\tau(\Omega)$ is well-defined and completely continuous for*

$$\tau = \min\{p^*/\alpha, p/\beta\}. \tag{9}$$

Proof. It is easily checked that (9) implies $(p^*)' < \tau \leq p$.

We define three operators

$$\begin{aligned}&I_2 : L^\tau(\Omega) \longrightarrow W^{-1,p'}(\Omega), \quad I_2(v) = v, \\ &I_1 : W^{-1,p'}(\Omega) \longrightarrow L^{p^*}(\Omega) \times L^p(\Omega, \mathbb{R}^n), \quad I_1(v) = \left(S^{-1}v, \nabla S^{-1}v\right), \\ &\Phi : L^{p^*}(\Omega) \times L^p(\Omega; \mathbb{R}^n) \longrightarrow L^\tau(\Omega), \quad \Phi(u,v) = g(\cdot, u, v).\end{aligned}$$

We observe that I_2 is completely continuous, since $L^\tau(\Omega) \hookrightarrow W^{-1,p'}(\Omega)$ is compact, and I_1 is continuous and bounded, because $\|u\|_{-1} \leq c_3\|u\|_\tau$. Further, Φ is continuous and bounded for $\tau = \min\{p^*/\alpha, p/\beta\}$. Indeed,

$$\begin{aligned}\|\Phi(u,v)\|_\tau^\tau &\leq \int_\Omega \left(3\max\left\{a_0|u|^\alpha, b_0|v|^\beta, |h|\right\}\right)^\tau dx \\ &\leq 3^\tau\left(a_0^\tau\|u\|_{\alpha\tau}^\alpha + b_0^\tau\|v\|_{\beta\tau}^\beta + \|h\|_\tau^\tau\right) \\ &\leq c\left(\|u\|_{p^*}^\alpha + \|v\|_p^\beta + \|h\|_\tau^\tau\right).\end{aligned}$$

Since g is a Carathéodory function, by using Lebesgue's dominated convergence theorem, we obtain the continuity of Φ.

Thus, $B = \Phi \circ I_1 \circ I_2 : L^\tau(\Omega) \longrightarrow L^\tau(\Omega)$ is a completely continuous operator. $\square$

Lemma 4 *Suppose that the hypotheses of Lemmas 2 and 3 are satisfied and, in addition, g satisfies (A_3). Then the set*

$$F = \{v \in L^\tau(\Omega) : v = Av + \lambda Bv, \quad \textit{for some } \lambda \in]0,1[\}$$

is bounded in $L^\tau(\Omega)$.

Proof. First, we will verify that the set of the solutions is bounded in $W^{-1,p'}(\Omega)$. Let $v \in F$. By Lemma 1, $v \in W^{-1,p'}(\Omega)$, and we have

$$\langle v, S^{-1}v\rangle = \langle Av, S^{-1}v\rangle + \lambda\langle Bv, S^{-1}v\rangle. \tag{10}$$

Now, by the properties of the operator S^{-1}, we have $\langle v, S^{-1}v\rangle = \|v\|_{-1}^{p/(p-1)}$, and hence, using (A_1) and (A_3), we can write

$$\begin{aligned}
\|v\|_{-1}^{p/(p-1)} =& \int_\Omega f\left(x, S^{-1}v, \nabla S^{-1}v, -v\right) S^{-1}v\,dx \\
&+ \int_\Omega g\left(x, S^{-1}v, \nabla S^{-1}v\right) S^{-1}v\,dx \\
&\le \int_\Omega \left(a|S^{-1}v|^{p-1} + b|\nabla S^{-1}v|^{p-1} + c|v| + |f(x,0,0,0)|\right)|S^{-1}v|\,dx \\
&+\sigma \int_\Omega |S^{-1}v|^p\,dx \\
&\le a\|S^{-1}v\|_p^p + b\|\nabla S^{-1}v\|_p^{p-1}\|S^{-1}v\|_{p'} + \|\gamma_0\|_{p'}\|S^{-1}(v)\|_p \\
&+c\int_\Omega |v||S^{-1}v|\,dx + \sigma\|S^{-1}v\|_p^p \\
&\le \left(\frac{a}{\lambda_1} + \frac{bc_\Omega}{\lambda_1^{1/p}} + \frac{\sigma}{\lambda_1}\right)\|S^{-1}v\|^p + c\|v\|_{-1}\|S^{-1}v\| + \frac{\|\gamma_0\|_p c_\Omega}{\lambda_1^{1/p}}\|S^{-1}v\| \\
&\le \left(\frac{a}{\lambda_1} + \frac{bc_\Omega}{\lambda_1^{1/p}} + c + \frac{\sigma}{\lambda_1}\right)\|v\|_{-1}^{p/(p-1)} + \frac{\|\gamma_0\|_p c_\Omega}{\lambda_1^{1/p}}\|v\|_{-1}^{1/(p-1)},
\end{aligned}$$

where $\gamma_0(x) = |f(x,0,0,0)|$. Then, from hypothesis (A_4),

$$\|v\|_{-1}^{p/(p-1)} \le \frac{\|\gamma_0\|_p c_\Omega}{\lambda_1^{1/p}}\|v\|_{-1}^{1/(p-1)}.$$

Therefore,

$$\|v\|_{-1} \le K_1,$$

where $K_1 = \|\gamma_0\|_p c_\Omega / \lambda_1^{1/p}$.

Finally, we will prove that $\|v\|_\tau \leq K$ for all $v \in F$ and $K > 0$. As $\alpha\tau \leq p^*$ and $\beta\tau \leq p$, we get

$$\begin{aligned}
\|B(v)\|_\tau^\tau = \|\Phi(S^{-1}v, \nabla S^{-1}v)\|_\tau^\tau \leq & C_0 \left(\|S^{-1}v\|_{\alpha\tau}^\alpha + \|\nabla S^{-1}v\|_{\beta\tau}^\beta + \|h\|_\tau^\tau \right) \\
\leq & \tilde{C}_0 \left(\|S^{-1}v\|^\alpha + \| \; S^{-1}v\|^\beta + \|h\|^\tau \right) \\
= & \tilde{C}_0 \left(\|v\|_{-1}^{\alpha/(p-1)} + \| \; v\|_{-1}^{\beta/(p-1)} + \|h\|_{-1}^{\tau/(p-1)} \right).
\end{aligned}$$

Hence, for $v \in F$, we have

$$\|v\|_\tau \leq \|A(v)\|_\tau + \lambda \|B(v)\|_\tau \leq l\|v\|_\tau + \gamma + K_2,$$

where $\gamma = \|f(.,0,0,0)\|_\tau$. This implies $\|v\|_\tau \leq K_2 + \gamma/(1-l)$, and the proof of this lemma is complete. □

Proof of Theorem 2 It follows at once from Lemmas 2–4 and Theorem 1. □

Finally, we would like to point out that the existence result for the implicit elliptic problem

$$u \in W_0^{1,p}(\Omega), \quad f(x, u, \nabla u, \triangle_p u) = 0,$$

obtained by Marino-Paratore [8] is a very special case of the result of this work.

Remark 2 It seems to be interesting to study a similar result for the implicit p-Kirchoff type problem

$$\begin{aligned}
-M \left(\int_\Omega |\nabla u|^p \, dx \right) \triangle_p u = f(x, u, \nabla u, \triangle_p u) + g(x, u, \nabla u) \quad \text{in } \Omega, \\
u = 0 \quad \text{on } \Gamma,
\end{aligned}$$

with $M : [0, +\infty) \to [m_0, +\infty)$, $m_0 > 0$, being a continuous function.

We plan to address these questions in a future research.

Acknowledgments. This research was partially supported by the Vicerectorado de Investigación-UNMSM-Perú, Proyecto de Investigación N^0. B24141981 R.R. N^0. 004305-R-24 and is part of the doctoral thesis of the second author.

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Please, cite to this paper as published in
Armen. J. Math., V. **16**, N. 12(2024), pp. 1–10
https://doi.org/10.52737/18291163-2024.16.12-1-10