

Fréchet Lie Algebroids and Their Cohomology

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Abstract. We define Lie and Courant algebroids on Fréchet manifolds. Moreover, we construct a Dirac structure on the generalized tangent bundle of a Fréchet manifold and show that it inherits a Fréchet Lie algebroid structure. We show that the Lie algebroid cohomology of the \mathcal{B} -cotangent bundle Lie algebroid of a weakly symplectic Fréchet manifold M is the Lichnerowicz-Poisson cohomology of M .

Key Words: Fréchet Lie algebroid, Dirac structures, Lichnerowicz-Poisson cohomology, Weak symplectic structures

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1 Introduction

In recent years Poisson geometry has been extended to the Banach manifolds context. In particular, the concept of Lie algebroid was generalized to the category of Banach vector bundles in [1, 11]. Lie algebroids were also defined on Projective limits of Banach Manifolds [4]. Dirac structures on Banach manifolds were studied in [2, 12]. The assertion that the Lie algebroid cohomology of the cotangent bundle Lie algebroid of a finite dimensional Poisson manifold M is the Lichnerowicz-Poisson cohomology of M was generalized to the Banach manifolds case in [7].

Our goal in this paper is to extend to Fréchet manifolds some of the aforementioned results. Fréchet manifolds arise in number of problems that have significance in global analysis and physical field theory. However, due to permanent problems with Fréchet spaces (i.e., problems of intrinsic nature) in most cases Fréchet manifolds are handled by indirect methods or only certain type of Fréchet manifolds are considered (see [6] for a survey on recent developments in Fréchet geometry). One of the main issues in the theory of Fréchet spaces is that the dual of a proper Fréchet space (not

Banachable) is never a Fréchet space. In addition, the space of continuous linear mappings of one Fréchet space to another is not a Fréchet space in general. This defect puts in question the way of defining cotangent bundle. In fact, as pointed out in [9], if M is a manifold modelled on a Fréchet space F , then in general there is no vector topology on the dual of F that can lead to a smooth manifold structure on the set-theoretic cotangent bundle of M . But the cotangent bundle of a Poisson manifold is a special case of Lie algebroid and vital to the Cartan calculus of differential forms. A way out of this difficulty was observed in [13], where a notion of the \mathcal{B} -cotangent bundle of a manifold modelled on a locally convex space was introduced and the Cartan calculus of differential forms was successfully adapted to the category of manifolds modelled on locally convex spaces.

By means of this notion we define Lie and Courant algebroids on Fréchet manifolds. Furthermore, we construct a Dirac structure as a subbundle of the generalized tangent bundle $TM'_\mathcal{B} \oplus TM$ of a Fréchet manifold M and show that it inherits a Lie algebroid structure from the Courant bracket. We show that a weak symplectic form of a Fréchet manifold M determines the so-called Lichnerowicz-Poisson cohomology of M and the Chevalley-Eilenberg cohomology of its \mathcal{B} -cotangent bundle Lie algebroid which are exactly the same.

We should mention that another approach to the geometry of Fréchet cotangent bundle is the use of the convenient setting which provides two different notions of cotangent bundles (kinematic and operational). We can attempt to use convenient calculus to develop Poisson geometry for Fréchet manifolds, but in this paper we consider only Micheal-Bastiani differentiability which is more familiar and applicable for people working in Fréchet spaces. It turns out by using the notion of \mathcal{B} -cotangent bundle most of the assertions and constructions are much the same as those of Banach manifolds case.

2 Preliminaries: Poisson Fréchet Manifolds

In this section, following [13], we define Poisson structures on Fréchet manifolds. We will apply the notion of differentiability in the Micheal-Bastiani sense. We will be working in the category of smooth manifolds and bundles.

Definition 1 *Let E and F be Fréchet spaces, $U \subseteq E$ open and $f : U \rightarrow F$ a continuous map. The derivative of f at $x \in U$ in the direction of $h \in E$ is defined as*

$$df(x)(h) := \lim_{t \rightarrow 0} \frac{1}{t} (f(x + ht) - f(x))$$

whenever the limit exists. The map f is called differentiable at x if $df(x)(h)$

exists for all $h \in E$. It is called a C^1 -map if it is differentiable at all points of U and

$$df : U \times E \rightarrow F, \quad (x, h) \mapsto df(x)(h)$$

is a continuous map. Higher directional derivatives and C^k -maps, $k \geq 2$, are defined in the obvious inductive fashion.

Within this framework Fréchet manifolds, Fréchet vector bundles (especially tangent bundles) and C^k -maps between Fréchet manifolds are defined in the obvious way (cf. [9]). However, for a manifold M modelled on a Fréchet space F we can define the set-theoretic cotangent bundle $T'M$ (without any topology on the fiber), but in general there is no vector topology on F' , the dual of F , that can lead to the identification $T'M \cong F \times F'$, see [9, Remark I.3.9]. Thus, we follow [13] and use the notion of the \mathcal{B} -cotangent bundle instead. In this definition to put a manifold structure on $T'M$, the dual of F is equipped by a \mathcal{B} -topology, where \mathcal{B} is a bornology on F . To be precise, we recall that a family \mathcal{B} of bounded subsets of F that covers F is called a bornology on F if it is directed upwards by inclusion and if for every $B \in \mathcal{B}$ and $r \in \mathbb{R}$ there is a $C \in \mathcal{B}$ such that $r \cdot B \subset C$.

Let E be a Fréchet space, \mathcal{B} a bornology on E and $L_{\mathcal{B}}(E, F)$ the space of all linear continuous maps from E to F . The \mathcal{B} -topology on $L_{\mathcal{B}}(E, F)$ is a Hausdorff locally convex topology defined by all seminorms $P_B^n(L) := \sup\{p_n(L(e)) \mid e \in B\}$, where $B \in \mathcal{B}$ and $\{p_n\}_{n \in \mathbb{N}}$ is a family of seminorms defining the topology of F . One similarly may define $L_{\mathcal{B}}^k(E, F)$ and $\bigwedge^k L_{\mathcal{B}}(E, \mathbb{R})$, the space of k -linear jointly continuous maps from E^k to F and the space of anti-symmetric k -linear jointly continuous maps from E^k to \mathbb{R} , respectively. If \mathcal{B} contains all compact sets, then the \mathcal{B} -topology on the space $L_{\mathcal{B}}(E, \mathbb{R}) = E'_{\mathcal{B}}$ of all continuous linear functionals on E , the dual of E , is the topology of compact convergence.

If \mathcal{B} contains all compact sets of E , then we define the differentiability of class $C_{\mathcal{B}}^k$: Let $U \subset E$ be open, a map $f : U \rightarrow F$ is called $C_{\mathcal{B}}^1$ if its partial derivatives exist and the induced map $df : U \rightarrow L_{\mathcal{B}}(E, F)$ is continuous. Similarly we can define maps of class $C_{\mathcal{B}}^k$, $k \in \mathbb{N} \cup \{\infty\}$, see [8, Definition 2.5.0]. A map $f : U \rightarrow F$ is $C_{\mathcal{B}}^k$, $k \geq 1$, if and only if f is C^k in the sense of Definition (1), see [8, Theorem 2.7.0 and Corollary 1.0.4 (2)]. In particular, f is $C_{\mathcal{B}}^{\infty}$ if and only if f is C^{∞} . Thus, if f at $x \in E$ is C^k and hence $C_{\mathcal{B}}^k$, then the derivative of f at x , $df(x)$, is an element of $E'_{\mathcal{B}}$.

Assume that \mathcal{B} is a bornology on F containing all compact sets and M is a Fréchet manifold modelled on F . Let f be a functional defined over M . The derivative of f at $x \in M$ can be written in terms of the iterated tangent bundles of M and we can consider $df : TM \rightarrow F$ given by $df(x, h) = df(x)(h)$ upon locally identifying TM with $U \times F$, where U is an open set in F . Therefore, if at $x \in M$ a map $f : M \rightarrow \mathbb{R}$ is C^k and hence $C_{\mathcal{B}}^k$, then $df(x)$ belongs to $L_{\mathcal{B}}(T_x M, \mathbb{R}) = (T_x M)'_{\mathcal{B}}$.

Definition 2 Let M be a Fréchet manifold modelled on a Fréchet space F and \mathcal{B} a bornology on F . The \mathcal{B} -cotangent bundle of M is defined as $TM'_\mathcal{B} := \bigcup_{x \in M} (T_x M)'_\mathcal{B}$ and the k -exterior product of the \mathcal{B} -cotangent bundle as $\bigwedge^k TM'_\mathcal{B} := \bigcup_{x \in M} \bigwedge^k (T_x M)'_\mathcal{B}$.

If \mathcal{B} is chosen such that $T(\mathcal{B}) \subset \mathcal{B}$ for all continuous linear endomorphisms T on F , then $\bigwedge^k TM'_\mathcal{B}$ is a vector bundle in the category of locally convex spaces with the local model $F \times \bigwedge^k F'_\mathcal{B}$. In particular, $TM'_\mathcal{B}$ is a vector bundle in the category of locally convex spaces with the local model $F \times F'_\mathcal{B}$, see [13, Remark (1), p. 339]. Therefore, in the sequel we always assume that bornologies have the mentioned property and contain all compact sets.

Let M be a manifold modelled on a Fréchet space F and \mathcal{B} a bornology on F . A smooth differentiable k -form of type \mathcal{B} is a smooth section of the bundle $\bigwedge^k TM'_\mathcal{B}$. We can also define a smooth differential k -form in the weak sense (which is usually used in the literature) as a section of the set-theoretic k -exterior bundle $\bigwedge^k TM'$, cf. [9]. A section ω of $\bigwedge^k TM'_\mathcal{B} \rightarrow M$ is a smooth differential k -form of type \mathcal{B} if and only if ω is a smooth differential k -form in the weak sense [13, Proposition IV.6]. Thus, in the sequel we call smooth differential forms of type \mathcal{B} simply smooth differential forms. We always assume that differential forms are smooth without mentioning it and write ω_x instead of $\omega(x)$ for $x \in M$.

We denote by $\mathfrak{X}(M)$ the space of all vector fields on M . The Lie bracket $[X, Y]$ of $X, Y \in \mathfrak{X}(M)$ is again vector field and $(\mathfrak{X}(M), [\cdot, \cdot])$ is a Lie algebra [9, Proposition II.3.7]. We further obtain for each $X \in \mathfrak{X}(M)$ and a k -form ω on M a unique linear smooth map $(\iota_X \omega)_x = \iota_{X(x)} \omega_x$, where $x \in M$ and $v, v_1, \dots, v_{k-1} \in T_x M$ and $(\iota_v \omega_x)(v_1, \dots, v_{k-1}) := \omega_x(v, v_1, \dots, v_{k-1})$, see [13, Lemma IV.7]. Let ω be a k -form on M . Then $d_{\text{dR}} \omega$, the de Rham derivative of ω , on vector fields $X_0, \dots, X_k \in \mathfrak{X}(M)$ is a smooth $(k+1)$ -form [13, Lemma IV.8] and is given by

$$\begin{aligned} (d_{\text{dR}} \omega)(X_0, \dots, X_k) &= \sum_{i=0}^{i=k} (-1)^i X_i(\omega(X_0, \dots, \hat{X}_i, \dots, X_k)) + \\ &\quad \sum_{0 \leq i < j \leq k} (-1)^{(i+j)} \omega([X_i, X_j], X_0, \dots, \hat{X}_i, \dots, \hat{X}_j, \dots, X_k), \end{aligned} \quad (1)$$

where a hat over symbols means omission. We now define the Lie derivative \mathcal{L}_X of a differential form in the direction of a vector field X by the Cartan formula:

$$\mathcal{L}_X = d_{\text{dR}} \circ \iota_X + \iota_X \circ d_{\text{dR}}. \quad (2)$$

Definition 3 A Fréchet manifold M is called weakly symplectic if for a

closed smooth 2-form ω on M the linear continuous map

$$\begin{aligned}\omega^\# : T_x M &\longrightarrow (T_x M)'_b \\ v_x &\mapsto \omega_x(v_x, \cdot) = \iota_{v_x} \omega_x\end{aligned}$$

is injective for all $x \in M$. Here, $(T_x M)'_b$ is the strong dual of the tangent space.

Definition 4 A vector field X_f on a weakly symplectic Fréchet manifold (M, ω) is called the symplectic gradient vector field of a smooth real valued function $f \in C^\infty(M)$ if $\mathrm{d}f = -\omega(X_f, \cdot) = \omega^\#(-X_f)$. Let $f, g \in C^\infty(M)$ be such that X_f and X_g exist. For f and g , the Poisson structure $\{f, g\}$ is defined by

$$\{f, g\} := \omega(X_f, X_g). \quad (3)$$

It is \mathbb{R} -linear, anti-symmetric and satisfies Jacobi identity, if all involved symplectic gradient vector fields exist. We say that a pair $\{M, \{\cdot, \cdot\}\}$ is a Fréchet Poisson manifold.

Remark 1 A Weak Poisson structure on a manifold modelled on a locally convex space can be defined without any need for a notion of cotangent bundle, see [10]. However, it is crucial to our purposes to have a subtle notion of cotangent bundle in order to define generalized tangent bundles, Lie algebroid structures and the LP-cohomology of a cotangent bundle.

3 Fréchet Lie Algebroids

Let M be a manifold modelled on a Fréchet space F and let $\pi : L \rightarrow M$ be a Fréchet vector bundle over M with fibers of type \mathbb{F} . We denote by $\Gamma(L)$ the space of all smooth sections of the vector bundle L . The spaces $\Gamma(L)$ and $\mathfrak{X}(M)$ are both $C^\infty(M)$ -modules.

Definition 5 A Lie algebroid L over M is a vector bundle $\pi : L \rightarrow M$ together with a bracket $[\cdot, \cdot]_L$ on the space $\Gamma(L)$ and a bundle map $\lambda_L : L \rightarrow TL$, called anchor, such that

1. The induced map $\lambda_L : (\Gamma(L), [\cdot, \cdot]_L) \rightarrow (\mathfrak{X}(M), [\cdot, \cdot])$ given by $(\lambda_L(s))(x) = \lambda_L(s(x))$, $x \in M$, $s \in \Gamma(L)$ is a Lie algebra homomorphism,
2. $[s_1, fs_2]_L = f[s_1, s_2]_L + \lambda_L(s_1)(f)s_2$ for every $f \in C^\infty(M)$ and $s_1, s_2 \in \Gamma(L)$.

The tangent bundle TM is trivially a Fréchet Lie algebroid for the usual Lie bracket of vector fields on M and the identity map of TM as an anchor map.

Definition 6 A Courant algebroid is a vector bundle $\pi : \mathbb{C} \rightarrow M$ together with an anchor $\lambda_{\mathbb{C}}$, a nondegenerate symmetric bilinear form Θ and a bracket $[\cdot, \cdot]_{\mathbb{C}}$ on $\Gamma(\mathbb{C})$ such that for all $s_1, s_2, s_3 \in \Gamma(\mathbb{C})$ and $f \in C^\infty(M)$

1. $[s_1, [s_2, s_3]_{\mathbb{C}}]_{\mathbb{C}} = [[s_1, s_2]_{\mathbb{C}}, s_3]_{\mathbb{C}} + [s_2, [s_1, s_3]_{\mathbb{C}}]_{\mathbb{C}}$,
2. $\lambda_{\mathbb{C}}(s_1)\Theta(s_2, s_3) = \Theta([s_1, s_2]_{\mathbb{C}}, s_3) + \Theta[s_2, [s_1, s_3]_{\mathbb{C}}]_{\mathbb{C}}$,
3. $[s_1, s_2]_{\mathbb{C}} + [s_2, s_1]_{\mathbb{C}} = \Delta(\Theta(s_1, s_2))$, where $\Delta : C^\infty(M) \rightarrow \Gamma(\mathbb{C})$ is defined by $\Theta(\Delta(f), s) = \lambda_{\mathbb{C}}(s)f$.

The \mathcal{B} -cotangent bundle $TM'_{\mathcal{B}}$ of M is a vector bundle in the category of locally convex spaces with the local model $F \times F'_{\mathcal{B}}$, therefore, the Whitney sum $TM \oplus TM'_{\mathcal{B}}$ makes sense. We denote by $\mathbb{T}M = TM \oplus TM'_{\mathcal{B}}$ the generalized tangent bundle of M . Let $X, Y \in \mathfrak{X}(M)$ and $\alpha, \beta \in \bigwedge^1 TM'_{\mathcal{B}}$. Now define the bracket $[X, Y]_{\mathbb{T}M} = ([X, Y], \mathcal{L}_X\beta - (\iota_Y \circ d_{\text{dR}} \alpha))$ and the anchor $\lambda_{\mathbb{T}M}$ given by $\lambda_{\mathbb{T}M}(X, \alpha) = X$. If we define $\Delta_{\mathbb{T}M}((X, \alpha), (Y, \beta)) = \alpha(Y) + \beta(X)$, then we can easily verify that for $(\mathbb{T}M, [X, Y]_{\mathbb{T}M}, \lambda_{\mathbb{T}M})$ the conditions (1)-(2) of the Definition 5 are fulfilled, therefore, $\mathbb{T}M$ is a Courant algebroid. The orthogonal complement \mathbb{L}^\perp of the subbundle $\mathbb{L} \subset \mathbb{T}M$ is defined as follows

$$\mathbb{L}^\perp := \{(X, \alpha) \in \mathbb{T}M : \Delta_{\mathbb{T}M}((X, \alpha), (Y, \beta)) = 0, \forall (Y, \beta) \in \mathbb{T}M\}.$$

Definition 7 A vector subbundle \mathbb{D} of the Courant algebroid $\mathbb{T}M$ that coincides with its orthogonal complement \mathbb{D}^\perp with respect to $\Delta_{\mathbb{T}M}$ is said to be an almost Dirac structure. It is called a Dirac structure if, in addition, is closed under the bracket $[\cdot, \cdot]_{\mathbb{T}M}$.

Define the Courant bracket on $\Gamma(\mathbb{T}M)$ by

$$\llbracket (X, \alpha), (Y, \beta) \rrbracket = \left([X, Y], \mathcal{L}_X\beta - \mathcal{L}_Y\alpha + \frac{1}{2} d_{\text{dR}}(\alpha(Y) - \beta(X)) \right).$$

We can easily show that the restriction of $\llbracket \cdot, \cdot \rrbracket$ to $\Gamma(\mathbb{D})$ yields a Lie bracket and if we let $\text{Pr} : \mathbb{D} \rightarrow TM$ be the restriction of the projection to TM , then $(\mathbb{D}, \llbracket \cdot, \cdot \rrbracket_{\mathbb{D}}, \text{Pr})$ is a Fréchet Lie algebroid.

4 Fréchet Lie Algebroids Cohomology

Let (M, ω) be a weakly symplectic Fréchet manifold. We denote by $\mathfrak{X}^k(M)$ and $\Omega^k(M)$ the spaces of all k -vector fields and k -differential forms on M , respectively. Define a morphism

$$\# \omega : \Omega^1(M) \rightarrow \mathfrak{X}^1(M); \beta(\# \omega(\alpha)) = \omega(\alpha, \beta), \forall \alpha, \beta \in \Omega^1(M). \quad (4)$$

The weak symplectic form ω induces a unique Lie bracket of 1-forms given by

$$\{\alpha, \beta\} = \mathcal{L}_{\# \omega(\alpha)} \beta - \mathcal{L}_{\# \omega(\beta)} \alpha - d_{\text{DR}} \omega(\alpha, \beta). \quad (5)$$

In general the existence of the Lie bracket is equivalent to the existence of the weak symplectic form ω , the proof is the same as the finite dimensional case, see [3]. Define the contravariant exterior differential $\sigma : \mathfrak{X}^k(M) \rightarrow \mathfrak{X}^{k+1}(M)$ for $X \in \mathfrak{X}^k(M)$ by

$$\begin{aligned} (\sigma X)(\alpha_0, \dots, \alpha_k) &= \sum_{i=0}^{i=k} (-1)^i \# \omega(\alpha_i) (X(\alpha_0, \dots, \hat{\alpha}_i, \dots, \alpha_k)) + \\ &+ \sum_{0 \leq i < j \leq k} (-1)^{(i+j)} X(\{\alpha_i, \alpha_j\}, \alpha_0, \dots, \hat{\alpha}_i, \dots, \hat{\alpha}_j, \dots, \alpha_k), \end{aligned} \quad (6)$$

where $\alpha_0, \dots, \alpha_k \in \Omega(M)$ and a hat over symbols means omission. Formally, the expression (6) is exactly the same as the de Rham derivative of forms and hence its algebraic consequences will be the same, in particular

- (i) $\sigma^2 = 0$,
- (ii) $\sigma(X_1 \wedge X_2) = \sigma(X_1) \wedge X_2 + (-1)^{\deg X_1} X_1 \wedge \sigma(X_2)$,
- (iii) $\sigma([X_1, X_2]) = -[\sigma(X_1), X_2] - (-1)^{\deg X_1} [X_1, \sigma(X_2)]$,

where $X_i (i = 1, 2)$ are k -vector fields on M and $\deg X_i$ is the degree of X_i . Therefore, $\Omega(M) := \bigoplus_{k \in \mathbb{N} \cup \{0\}} (\Omega^k M)$ with the coboundary operator σ is a cochain complex and we can define the Lichnerowicz-Poisson cohomology of M .

Definition 8 $\Omega(M)$ with the coboundary operator σ is called the Lichnerowicz-Poisson cochain of M , and

$$\mathbf{H}_{LP}^k(M, \omega) := \frac{\ker \left(\mathfrak{X}^k(M) \xrightarrow{\sigma} \mathfrak{X}^{k+1}(M) \right)}{\text{im} \left(\mathfrak{X}^{k-1}(M) \xrightarrow{\sigma} \mathfrak{X}^k(M) \right)} \quad (7)$$

are Lichnerowicz-Poisson cohomology or LP-cohomology spaces of M .

The following LP-cohomology spaces can be computed directly by Definition 8.

Let $Z_{\{C^\infty(M), \{\cdot, \cdot\}\}} = \{f \in C^\infty(M) : \forall g \in C^\infty(M), X_g f = 0\}$ and let $\mathfrak{X}_S^1(M)$ be the space of symplectic gradient vector fields $X_f, f \in C^\infty(M)$. Let $\mathfrak{X}_\omega^1(M) := \{X \in \mathfrak{X}(M) : \mathcal{L}_X \omega = 0\}$. We then have

- (i) $\mathbf{H}_{LP}^0(M, \omega) = Z_{\{C^\infty(M), \{\cdot, \cdot\}\}}$, since $\sigma f = -X_f$.

$$(ii) \mathbf{H}_{LP}^1(M, \omega) = \frac{\mathfrak{X}_\omega^1(M)}{\mathfrak{X}_S^1(M)}.$$

Now we define the Chevalley-Eilenberg cohomology [5] associated to Fréchet Lie algebroids. Let $(L, \lambda_L, [\cdot, \cdot]_E)$ be a Fréchet Lie algebroid over a Fréchet manifold M . Let $C^\infty(M)$ act on $\Gamma(L)$ by $(s, f) \mapsto \lambda_L(s)f$. A k -linear anti-symmetric mapping $\ell_k : \Gamma(L)^k \rightarrow C^\infty(M)$ is called a $C^\infty(M)$ -valued k -cochain. Let $C^k(\Gamma(L); C^\infty(M))$ be the vector space of these cochains. Define the operator d_L by

$$\begin{aligned} (d_L \ell_k)(s_0, \dots, s_k) &= \sum_{i=0}^{i=k} (-1)^i \lambda_L(s_i) (\ell_k(s_0, \dots, \hat{s}_i, \dots, s_k)) + \\ &+ \sum_{0 \leq i < j \leq k} (-1)^{(i+j)} \ell_k([s_i, s_j]_L, s_0, \dots, \hat{s}_i, \dots, \hat{s}_j, \dots, s_k), \end{aligned}$$

for a k -cochain ℓ_k and $s_0, \dots, s_k \in \Gamma(L)$. Like the case of the De Rham derivative of forms we obtain $d_L \circ d_L = 0$. Therefore, $C^k(\Gamma(L); C^\infty(M))$ with d_L forms a Chevalley-Eilenberg cochain and the corresponding cohomology spaces

$$\mathbf{H}^k(C^k(\Gamma(L); C^\infty(M))) := \frac{\ker \left(\Gamma(L)^k \xrightarrow{d_L} \Gamma(L)^{k+1} \right)}{\text{im} \left(\Gamma(L)^{k-1} \xrightarrow{d_L} \Gamma(L)^k \right)},$$

are called the Lie algebroid cohomology of $\Gamma(L)$ with coefficient in $C^\infty(M)$. For the tangent bundle Lie algebroid TM of M , the Lie algebroid cohomology is just the De Rham cohomology of M .

Theorem 1 *The Lie algebroid cohomology of the \mathcal{B} -cotangent bundle Lie algebroid is the Lichnerowicz-Poisson cohomology of M .*

Proof. On any weakly symplectic Fréchet manifold (M, ω) the bracket $\{\cdot, \cdot\}$ of 1-forms (5) defines a Lie algebroid structure on the \mathcal{B} -cotangent bundle $TM'_\mathcal{B}$ with the anchor $\#\omega : TM'_\mathcal{B} \rightarrow TM$ given by $\beta(\#\omega(\alpha)) = \omega(\beta, \alpha)$; $\alpha, \beta \in TM'_\mathcal{B}$. In this case, eventually the operator $d_{(TM'_\mathcal{B})}$ coincides with the contravariant exterior differential (6) and so the cohomologies coincide \square

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