

Arshag Hajian: By the 85th Anniversary

This issue of the Journal is dedicated to the well-known American mathematician A. Hajian who made many original contributions to Ergodic Theory.

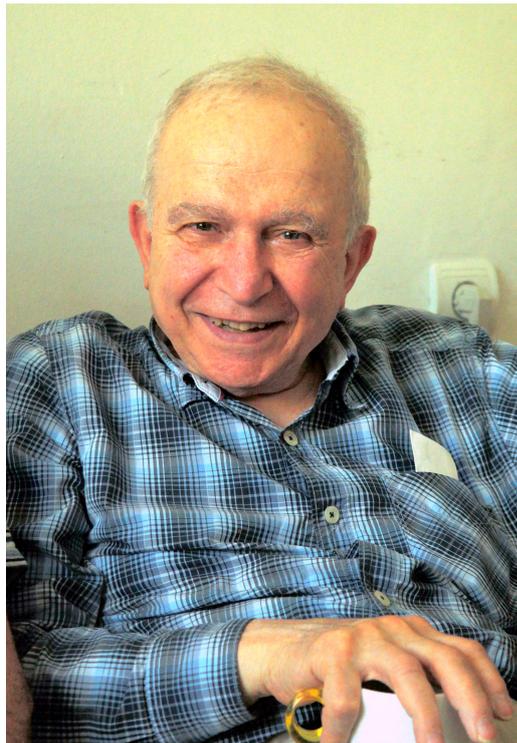
Arshag Berj Hajian was born in 1930 in Cairo, the capital of Egypt. During the Genocide, this country sheltered many Armenian refugees, who miraculously escaped from the Turkish scimitar. His father, while still being a teen-ager was among the more than 4,500 heroic participants of the 53-day defense of Musa Dagh who were evacuated to Port Said by the Allies warships after people have been discovered on the coast from the French armoured cruiser *Guichen*.

Unlike many of his peers who had chosen the path of artisans and merchants, young Arshag while in college turned to studying science. In the early 1950's he decided to continue his studies in the United States where in the post-war years many famous scientists taught.

He was at first accepted into the Electrical Engineering department of Pennsylvania State University. A year later he moved to the Faculty of Physics of the University of Chicago which in those years taught many famous scientists such as L. Szilard, E. Fermi, E. Teller, and S. Chandrasekhar.

Finally he transferred to the Department of Mathematics. Here again he was in the company of distinguished scientists: A. Weil, M.H. Stone, P. Halmos, I. Kaplanski, I. Segal, Sh.-Shen Chern, A. Zygmund, S. Mac Lane, most of whom were his lecturers. In addition he also had the opportunity to meet and talk with C.H. Hille, N. Jacobson, and H. Helson.

At last fate brought him to Yale University and one of the most prominent mathematicians of the time, Shizuo Kakutani - a man with whom his research would be forever linked.



Among other things, Kakutani was interested in Ergodic Theory, the foundations of which in connection with the needs of statistical physics in the 1930's were laid by von Neumann, Birkhoff and Hopf and which in turn rested upon the earlier remarkable Poincaré recurrence Theorem, Liouville's Theorem in classical mechanics and the treatises by Boltzmann. The beginning of the second half of the century was the heyday of the abstract theory.

Professor Kakutani directed Arshag toward the well-known work of E. Hopf on ergodic theory. Arshag immediately acquired a taste for research in the area of infinite measure spaces and this became the main topic of his future activity. Over time he has become one of the leading specialists in the theory of transformations preserving an infinite measure.

In his thesis, Arshag introduced the dual concepts of weakly wandering sets and sequences into the field of Ergodic theory surprising many who had no inkling of their existence.

In collaboration with Kakutani he continued investigating weakly wandering sets and sequences and then went on to introduce and study exhaustive weakly wandering sets and sequences. Together they constructed a number of examples of ergodic measure preserving transformations in spaces with infinite measure which were completely contrary to the intuition of mathematicians accustomed to working in probability spaces and revealing a serious difference between the properties of measure preserving ergodic transformations in spaces with finite measure and those with infinite measure (see the survey in [1]). One of the most well known of these examples is now referred to as the Hajian-Kakutani transformation.

Arshag continued his studies and a deep relation with problems in Number Theory was discovered. Specifically the arithmetic properties of the exhaustive weakly wandering sequences which arise in ergodic theory is intimately connected to complementary subsets of the integers which is studied in number theory.

Throughout his life Arshag has kept a deep respect and affection for his teacher, always speaking about him in the most sublime expressions. Over time, a group of enthusiasts have appeared which further developed the theory of ergodic transformations in spaces with infinite measure. This includes a group of longterm co-authors and colleagues (especially Stanley Eigen, Yuji Ito and Vidhu Prasad). In the past year these authors in collaboration with A. Hajian published in *Springer* a monograph *Weakly Wandering Sequences in Ergodic Theory*. The book also was inspired by a desire of Kakutani (who died before his idea was embodied in life) to create a work summarizing decades of researches in this area. The monograph is dedicated to the memory of their teacher.

For many years Arshag Hajian worked at the Northeastern University in Boston, and only this year he retired.

In addition to his fruitful mathematical activity, Arshag has proved himself as a true patriot of Armenia. He repeatedly came to Soviet Armenia, and his visits to

independent Armenia acquired a systematic character. His son Aram, the head of the engineering department at the American University of Armenia was born in the USA and now lives in Armenia. Arshag bought a house in Yerevan where he stays during visits, and currently his daughter also has an apartment in Yerevan.

In the hungry and cold 90s, he tirelessly fought for the survival of mathematical research in Armenia. Everyone remembers how overcoming the obstacles created by the officials he brought to the Institute of Mathematics the components of the first personal computers and collected them in the unheated rooms. On his initiative and at his own expenses the Research Mathematics Fund was established, which played a crucial role in the continuation of mathematical researches in the country, providing scholarships to the best students, paying for scientific trips of our scientists and the receptions of foreign experts. For over 20 years the fund paid on behalf of the Armenian Mathematical Union (in the activity of which he also takes an important part) the quite large annual fees to the International Mathematical Union. The election of Arshag as a foreign member of the National Academy of Sciences is a minimum act of recognition of his services to Armenian science.

In spite of a hard life and disease he is always cheerful and active, friendly to all, and infusing optimism into everyone who has the good fortune to chat with him. At the same time, he continues the traditions of his ancestors, which, thanks to the spirit and will to win have saved themselves and their families (as written in 1916 by Lord Bryce in *The Treatment of Armenians in the Ottoman Empire 1915-16*, the Musa Dagh epic *is the only story ... with a happy ending*).

The main lesson he teaches others is to abstract from the difficulties of routine life and distressing reality, and focus on how to preserve the scientific potential of Armenia, the loss of which would turn the country into an apricot republic.

We wish Prof. Hajian long life and fruitful activity in all fields, because the rest, even the well-deserved, is completely contraindicated in him.

Happy anniversary, dear Arshag!

V. Arzumanian, R. Barkhudaryan, S. Eigen, A. Nersessian

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