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The Convergence Acceleration of Two-Dimensional Fourier Interpolation

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Abstract. Hereby, the convergence acceleration of two-dimensional trigonometric interpolation for a smooth functions on a uniform mesh is considered. Together with theoretical estimates some numerical results are presented and discussed that reveal the potential of this method for application in image processing. Experiments show that suggested algorithm allows acceleration of conventional Fourier interpolation even for sparse meshes that can lead to an efficient image compression/decompression algorithms and also to applications in image zooming procedures.

Key Words: Convergence Acceleration, Interpolation, Local Fourier Analysis, Image Processing
Mathematics Subject Classification 2020: 41A60, 68U10

Introduction

It is well known that Fourier series and Fourier interpolation are powerful tools for theoretical and applied investigations. The main drawback that diminishes their strength is the Gibbs phenomenon near the points of singularities of the approximated function.

Let f be a piecewise smooth function on $[-1,1]$ with jump points $\{a_k\}$, $-1 = a_0 < a_1 < \cdots < a_{l-1} < 1$, $2 \leq l < \infty$. Suppose that $f \in C^{q+1}$, $q \geq 0$ on each segment $[a_k, a_{k+1}]$, $k = 1, \cdots, l-2$ and also on the segments $[-1, a_1]$, $[a_{l-1}, 1]$.

Denote by

$$A_{sk} = f^{(k)}(a_s + 0) - f^{(k)}(a_s - 0),\ k = 0, \cdots, q;\ s = 1, \cdots, l-1,$$
$$A_{0k} = f^{(k)}(-1) - f^{(k)}(1),\ k = 0, \cdots, q$$

the jumps of f and its derivatives at the points $\{a_s\}$. By $\{f_n\}$ we denote Fourier coefficients of f

$$f_n = \frac{1}{2}\int_{-1}^{1} f(t)e^{-i\pi nt}dt,\ n = 0, \pm1, \cdots. \qquad (1)$$

1 Two-dimensional acceleration

1.1 Basic Notations

For $f \in C^{2q}[-1,1]^2$ denote

$$f^{(k,s)}(x,y) = \frac{\partial^{k+s} f(x,y)}{\partial x^k \partial y^s},\ k,s = 0,\cdots,q,$$

$$u_k(y) = f^{(k,0)}(1,y) - f^{(k,0)}(-1,y),\ k = 0,\cdots,q,$$
$$v_k(x) = f^{(0,k)}(x,1) - f^{(0,k)}(x,-1),\ k = 0,\cdots,q,$$
$$\Delta_{k,s} = f^{(k,s)}(1,1) - f^{(k,s)}(-1,1) - f^{(k,s)}(1,-1) + f^{(k,s)}(-1,-1),\ k,s = 0,\cdots,q.$$

By f_{nm}, $u_{k,m}$ and $v_{s,n}$ we denote the following Fourier coefficients

$$f_{nm} = \frac{1}{4}\int_{-1}^{1}\int_{-1}^{1} f(x,y) e^{-i\pi(nx+my)} dx dy,$$

$$u_{k,m} = \frac{1}{2}\int_{-1}^{1} u_k(y) e^{-i\pi m y},\ v_{k,n} = \frac{1}{2}\int_{-1}^{1} v_k(x) e^{-i\pi n x},\ k = 0,\cdots,q.$$

The next two lemmas can easily be proved by means of integration by parts (see [4]).

Lemma 1 *For any $f \in C^{2q+2}[-1,1]^2$, $q \geq 0$ the following formula holds for $n, m \neq 0$*

$$\begin{aligned} f_{nm} = {} & \frac{(-1)^{n+1}}{2}\sum_{k=0}^{q}\frac{u_{k,m}}{(i\pi n)^{k+1}} + \frac{(-1)^{m+1}}{2}\sum_{s=0}^{q}\frac{v_{s,n}}{(i\pi m)^{s+1}} - \\ & -\frac{(-1)^{n+m}}{4}\sum_{s=0}^{q}\sum_{k=0}^{q}\frac{\Delta_{k,s}}{(i\pi n)^{k+1}(i\pi m)^{s+1}} + \\ & +\frac{1}{4(i\pi n)^{q+1}(i\pi m)^{q+1}}\int_{-1}^{1}\int_{-1}^{1} f^{(q+1,q+1)}(t,z) e^{-i\pi(nt+mz)} dz dt. \end{aligned} \tag{2}$$

Theorem 1 *[4] . If $f \in C^{2q+4}([-1,1]\times[-1,1])$, $q \geq 0$ then*

$$\lim_{N\to\infty}(2N+1)^{q+2}||f - S_{q,N}(f)||^2 = \frac{2^{2q+3}}{(2q+3)\pi^{2q+4}}\int_{-1}^{1}\left(|\widetilde{\varphi}_q(x)|^2 + \left|\widetilde{\psi}_q(x)\right|^2\right) dx,$$

where

$$\widetilde{\varphi}_q(y) = \int_{-1}^{1} B_q(t)\varphi_q(y-t)dt,\ \widetilde{\psi}_q(x) = \frac{1}{2}\int_{-1}^{1} B_q(t)\psi_q(x-t)dt.$$

$$\varphi_q(y) = f^{(q+1,q+1)}(1,y) - f^{(q+1,q+1)}(-1,y),$$

$$\psi_q(x) = f^{(q+1,q+1)}(x,1) - f^{(q+1,q+1)}(x,-1)$$

and

$$\|f\| = \left(\int_{-1}^{1} |f(x)|^2 dx \right)^{1/2}.$$

Proof. Using (2) and (1) □

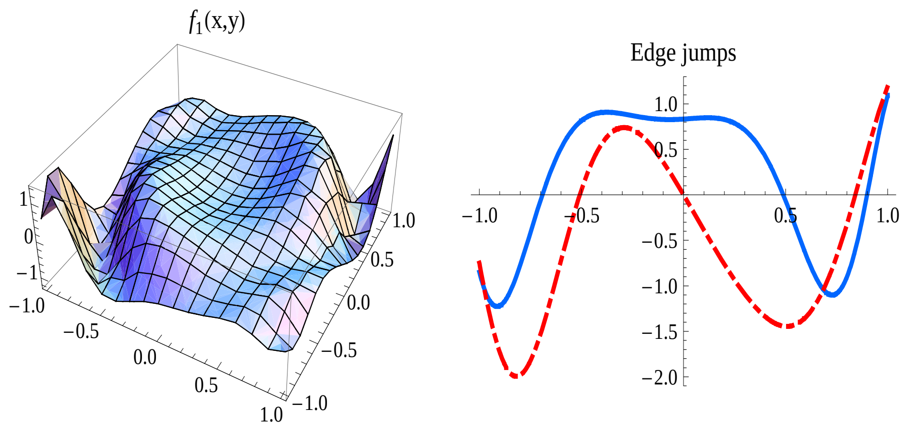

Figure 1: Graph of the function $f_1(x,y)$ values (left) as well as (right) values of $f_1(x,1) - f_1(x,-1)$ (blue) and $f_1(1,y) - f_1(-1,y)$ (red-dashing), $-1 \leq x, y \leq 1$

Sample

Equation references: (1), (2)

Theorems, lemmas, sections references: Lemma 1, Theorem 1, Section 1.1, Figure 1

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